Options Risk Calculator

Technical Formula Reference

English

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Fundamentals: Discounting and Underlyings

Continuous Discount Factor

The calculator uses *Continuous Compounding* throughout. The risk-free discount factor for maturity T (in years) and interest rate r:

$$B(0,T) = e^{-rT}$$

This is consistent with Black-Scholes and the CRR binomial tree. The interest rate r is treated as continuously compounded – not as an annual discrete rate.

Spot Price per Underlying

Underlying	Spot S	Special feature
Stock	Stock price	—
Index	Index level	no dividend yield
FX	Exchange rate	Standard BS (no foreign rate)
Commodity	Spot price	—
Zero-Coupon Bond	$S = N \cdot e^{-yT}$	N = face value, y = yield (continuous)

Zero-Coupon Bond

The fair price of a ZCB with face value N, yield y (continuous) and remaining maturity T:

$$S_{\text{ZCB}} = N \cdot e^{-yT}$$

This value is used as spot price S in Black-Scholes. σ denotes the price volatility of the bond.

Pricing Model 1: Black-Scholes (BS)

Assumptions

- Geometric Brownian motion: $dS = \mu S\,dt + \sigma S\,dW_t$
- Constant volatility σ and interest rate r
- No dividends (standard formulation)
- European option (exercise only at expiry)
- No transaction costs; short selling permitted

Auxiliary Quantities d_1 and d_2

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \qquad d_2 = d_1 - \sigma\sqrt{T}$$

$\Phi(\cdot)$ denotes the cumulative standard normal distribution, $\phi(\cdot)$ the standard normal density.

Option Prices

$$C = S\,\Phi(d_1) - K\,e^{-rT}\,\Phi(d_2) \tag{1}$$

$$P = K\,e^{-rT}\,\Phi(-d_2) - S\,\Phi(-d_1) \tag{2}$$

Put-call parity: $C - P = S - K\,e^{-rT}$.

Greeks (Black-Scholes)

Greek	Symbol	Call	Put
Delta	Δ	$\Phi(d_1)$	$\Phi(d_1) - 1$
Gamma	Γ	$\dfrac{\phi(d_1)}{S\,\sigma\sqrt{T}}$	
Theta	Θ	$-\dfrac{S\phi(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}\Phi(d_2)$	$-\dfrac{S\phi(d_1)\sigma}{2\sqrt{T}} + rKe^{-rT}\Phi(-d_2)$
Vega	ν	$S\,\phi(d_1)\,\sqrt{T}$	
Rho	ρ	$KTe^{-rT}\Phi(d_2)$	$-KTe^{-rT}\Phi(-d_2)$

Note: Theta is output per calendar day (divided by 365). Vega and Rho refer to a change of $+1\%$ absolute in σ and r respectively.

Pricing Model 2: Binomial Tree (CRR)

Parametrisation (Cox-Ross-Rubinstein)

For N steps, time step $\Delta t = T/N$:

$$u = e^{\sigma\sqrt{\Delta t}}, \qquad d = \frac{1}{u}, \qquad p = \frac{e^{r\,\Delta t} - d}{u - d}$$

p is the risk-neutral up-probability. The calculator uses $N = 100$.

Backward Induction

Terminal node price with i down-moves: $S_{N,i} = S \cdot u^{N-i} \cdot d^{i}$. Option value at terminal nodes:

$$V_{N,i} = \max(S_{N,i} - K,\ 0) \text{ (Call)}, \qquad V_{N,i} = \max(K - S_{N,i},\ 0) \text{ (Put)}$$

$$V_{j,i} = e^{-r\,\Delta t}\left[p \cdot V_{j+1,i} + (1-p) \cdot V_{j+1,i+1}\right]$$

Option price: $V_{0,0}$.

Greeks (Approximation from First/Second Tree Step)

$$\Delta \approx \frac{V_{1,0} - V_{1,1}}{S(u-d)}, \qquad \Gamma \approx \frac{V_{2,0} - 2V_{2,1} + V_{2,2}}{\left[\frac{1}{2}(Su^2 - Sd^2)\right]^2 / S}$$

Theta, Vega, Rho are computed via Black-Scholes approximation at the boundary node.

Option Types and Payoffs

All spreads are priced as combinations of vanilla options. Price and Greeks are additive.

Vanilla Call and Put

Type	Payoff at expiry	Price
Call	$\max(S_T - K,\ 0)$	$C(S, K, T, r, \sigma)$ per (1)
Put	$\max(K - S_T,\ 0)$	$P(S, K, T, r, \sigma)$ per (2)

Bull Call Spread ($K_1 < K_2$)

$$\text{Payoff} = \max(S_T - K_1, 0) - \max(S_T - K_2, 0)$$
$$\text{Price} = C(S, K_1) - C(S, K_2)$$

Max. gain: $K_2 - K_1 -$ Premium. Max. loss: Premium.

Bear Put Spread ($K_1 < K_2$)

$$\text{Payoff} = \max(K_2 - S_T, 0) - \max(K_1 - S_T, 0)$$
$$\text{Price} = P(S, K_2) - P(S, K_1)$$

Butterfly Spread ($K_1 < K_2 < K_3$, typically $K_2 = \frac{K_1+K_3}{2}$)

$$\text{Payoff} = \max(S_T - K_1, 0) - 2\,\max(S_T - K_2, 0) + \max(S_T - K_3, 0)$$
$$\text{Price} = C(S, K_1) - 2\,C(S, K_2) + C(S, K_3)$$

Max. gain at $S_T = K_2$: $(K_2 - K_1) - \text{Premium}$.

Risk Measures: VaR and Break-Even

Monte Carlo Simulation

The calculator simulates $N = 20\,000$ paths:

$$S_T^{(k)} = S \cdot \exp\Big[\big(r - \tfrac{1}{2}\sigma^2\big)T + \sigma\sqrt{T}\,Z^{(k)}\Big], \quad Z^{(k)} \sim \mathcal{N}(0,1)$$

P&L per path: $\pi^{(k)} = \text{Payoff}(S_T^{(k)}) - \text{Premium}$.

Value at Risk

After sorting $\pi^{(1)} \le \cdots \le \pi^{(N)}$:

$$\text{VaR}_{95\%} = -\pi^{(\lfloor 0.05\,N \rfloor)}, \qquad \text{VaR}_{99\%} = -\pi^{(\lfloor 0.01\,N \rfloor)}$$

A positive VaR value corresponds to a loss.

Break-Even

Option type	Break-even
Call (Vanilla)	$K_1 + \text{Premium}$
Put (Vanilla)	$K_1 - \text{Premium}$
Bull Call Spread	$K_1 + \text{Premium}$
Bear Put Spread	$K_2 - \text{Premium}$
Butterfly	K_2 (approximation)

Worked Example: FX Option (EUR/USD), Call

Input Parameters

Parameter	Symbol	Value
Underlying	—	EUR/USD
Option type	—	European Call
Spot	S	1.0800
Strike	K	1.0800 (at-the-money)
Volatility	σ	8.00%
Interest rate (continuous)	r	3.00%
Maturity	T	90 days = 90/365 = 0.24658 years

Step 1: Auxiliary Quantities

Discount factor:

$$e^{-rT} = e^{-0.03\times 0.24658} = e^{-0.007397} = 0.99263$$

$$d_1 = \frac{\ln(1) + (0.03 + 0.0032) \times 0.24658}{0.08 \times 0.49657} = \frac{0.03320 \times 0.24658}{0.039726} = \frac{0.008187}{0.039726} = 0.2061$$

$$d_2 = 0.2061 - 0.08 \times 0.49657 = 0.2061 - 0.03973 = 0.1664$$

Normal distribution values:

$$\Phi(0.2061) = 0.5817, \qquad \Phi(0.1664) = 0.5661$$

Step 2: Black-Scholes Price

$$\begin{aligned} C_{\text{BS}} &= S\,\Phi(d_1) - K\,e^{-rT}\,\Phi(d_2) \\ &= 1.0800 \times 0.5817 - 1.0800 \times 0.99263 \times 0.5661 \\ &= 0.62824 - 0.60716 = \mathbf{0.02108} \end{aligned}$$

Price: **0.0211 USD** per 1 EUR notional (approx. 211 USD for a 10,000 EUR lot).

Step 3: Greeks (Black-Scholes)

Normal density: $\phi(0.2061) = 0.39130$

Greek	Calculation	Result	Interpretation
Δ	$\Phi(0.2061)$	$+0.5817$	Price $+0.582$ per $+1$ spot
Γ	$\dfrac{0.39130}{1.08 \times 0.08 \times 0.49657}$	$+9.095$	Delta change per $+1$ spot
Θ	$-\dfrac{1.08 \times 0.39130 \times 0.08}{2 \times 0.49657} - 0.03 \times 1.08 \times 0.99263 \times 0.5661$	-0.000456	Loss per calendar day
ν	$1.08 \times 0.39130 \times 0.49657$	$+0.2096$	Price per $+1\%$ vol
ρ	$\dfrac{1.08 \times 0.24658 \times 0.99263 \times 0.5661}{100}$	$+0.001490$	Price per $+1\%$ rate

Step 4: Binomial Tree (CRR, $N = 100$)

$$\Delta t = \frac{0.24658}{100} = 0.0024658, \qquad u = e^{0.08\times\sqrt{0.0024658}} = e^{0.003973} = 1.003981$$

$$d = \frac{1}{1.003981} = 0.996035, \qquad p = \frac{e^{0.03\times 0.0024658} - 0.996035}{1.003981 - 0.996035} = \frac{0.004039}{0.007946} = 0.50831$$

Backward induction over 100 steps yields:

$$C_{\text{Bin}} \approx \mathbf{0.02107}$$

Deviation from BS: $< 0.005\%$ – expected convergence of the CRR tree at $N = 100$.

Step 5: VaR (Monte Carlo, $N = 20\,000$)

Metric	Result
$\text{VaR}_{95\%}$	0.0211 (total loss of premium)
$\text{VaR}_{99\%}$	0.0211
Break-even	$1.0800 + 0.0211 = \mathbf{1.1011}$

Explanation: An ATM option expires worthless in approx. 42% of paths. Since the loss is capped at the premium, the VaR equals the total loss of premium.

Step 6: P&L Scenarios

EUR/USD at T	Δ Spot	Payoff	Net P&L	Return
0.7560	−30%	0.0000	−0.0211	−100.0%
0.8640	−20%	0.0000	−0.0211	−100.0%
0.9720	−10%	0.0000	−0.0211	−100.0%
1.0260	−5%	0.0000	−0.0211	−100.0%
1.0800	±0%	0.0000	−0.0211	−100.0%
1.1340	+5%	0.0540	+0.0329	+156.2%
1.1880	+10%	0.1080	+0.0869	+412.3%
1.2960	+20%	0.2160	+0.1949	+924.8%
1.4040	+30%	0.3240	+0.3029	+1437.0%

Output Reference

Output	Calculation	Model
Option price	BS formula or CRR tree	BS / Bin
e^{-rT} (discount factor)	continuous, throughout	both
Break-even	$K \pm$ Premium	—
$\Delta, \Gamma, \Theta, \nu, \rho$	analytical (BS) / numerical (Bin)	both
$\text{VaR}_{95\%/99\%}$	Monte Carlo, $N = 20\,000$	—
P&L scenarios	$\pm 5\%, 10\%, 20\%, 30\%$	—
CSV export	all fields above	—

All formulas correspond to the JavaScript implementation in options-risk-calculator.html (v1.1). Deviations between BS and Binomial Tree are typically below 0.01% at $N = 100$.