Forward Swap VaR Calculator

Mathematical Foundations

English

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Models and formulas of the calculator: yield curve and interpolation, valuation of the forward starting swap, sensitivities (DV01, duration, convexity) and parametric Value at Risk based on historical discount factor time series. Companion document: *User Manual with Worked Example*.

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Table of Contents

1 Overview **2**
1.1 Notation . . . 2

2 Yield Curve **2**
2.1 Zero rates and discount factors . . . 2
2.2 Interpolation of the current curve . . . 3
2.3 Interpolation of the historical curves . . . 3

3 Valuation of the Forward Starting Swap **3**
3.1 Payment schedule . . . 3
3.2 Forward rates . . . 3
3.3 Cash flows and present value . . . 4
3.4 Closed form . . . 4

4 Sensitivities **4**
4.1 DV01 . . . 4
4.2 Modified duration . . . 4
4.3 Convexity . . . 5

5 Value at Risk **5**
5.1 Historical discount factor time series . . . 5
5.2 Log returns . . . 5
5.3 Covariance and correlation matrix . . . 6
5.4 Sensitivity vector . . . 6
5.5 Portfolio variance and VaR . . . 6
5.6 Inverse normal distribution . . . 6

6 Model Assumptions and Limitations **7**
6.1 Assumptions . . . 7
6.2 Comparison with frozen net cash flows . . . 7
6.3 Further points . . . 7

7 Mapping Formula – Code **8**

1 Overview

The calculator values a *forward starting interest rate swap* and measures its market risk. It considers a **payer swap**: the holder pays the fixed rate K and receives the floating rate. The swap starts at time T_0 (forward start) and runs for T years.

The calculation consists of five steps (Figure 1):

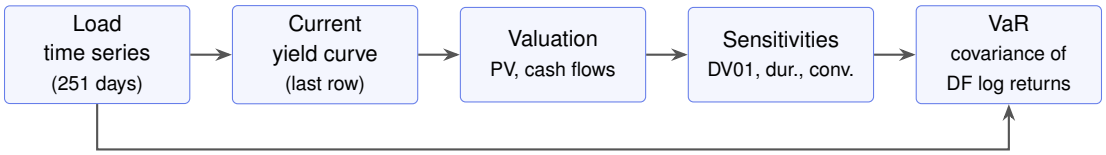


Figure 1: Calculation flow of the Forward Swap VaR Calculator

1.1 Notation

Symbol	Meaning	Code
N	notional amount in EUR	`notional`
K	fixed rate (decimal, p. a.)	`fixedRate`
T_0	forward start in years	`forwardStart`
T	tenor (term) in years	`tenor`
f	payment frequency per year $(1, 2, 4)$	`frequency`
$\delta = 1/f$	length of an accrual period in years	`paymentInterval`
n	number of payments	`nPayments`
t_i	payment dates, $i = 1, \ldots, n$	`paymentDates`
τ_j, r_j	curve pillars (tenors) and zero rates	`yieldCurveTenors, yieldCurve`
$r(t)$	interpolated zero rate for maturity t	`interpolateRate`
$P(t)$	discount factor for maturity t	`discountFactor`
F_i	simple forward rate of period i	`forwardRates`
C_i	net cash flow of period i	`netCashflows`
α	confidence level	`confidence`
h	time horizon in trading days	`horizon`

All times are treated as year fractions; no calendar or day count convention is applied.

2 Yield Curve

2.1 Zero rates and discount factors

The yield curve is given by m pillars $(\tau_j, r_j)_{j=1,\ldots,m}$. The rates are interpreted as **continuously compounded zero rates**. The discount factor for maturity t is

$$P(t) = e^{-r(t)\,t}. \tag{1}$$

Implementation: discountFactor(rate, time)

```
return Math.exp(-rate * time);
```

The time series comes from a CSV file or from one of the two sample series built into the calculator. The *current* yield curve is the last row of the loaded time series. The file contains rates in percent; they are divided by 100 on import.

2.2 Interpolation of the current curve

For valuation, $r(t)$ is determined by `interpolateRate(t)`:

$$r(t) = \begin{cases} r_1, & t \le \tau_1, \\ r_j + \dfrac{r_{j+1} - r_j}{\tau_{j+1} - \tau_j}\,(t - \tau_j), & \tau_j \le t \le \tau_{j+1}, \\ r_m, & t \ge \tau_m. \end{cases} \tag{2}$$

This is linear interpolation in zero rates with flat extrapolation at both ends.

2.3 Interpolation of the historical curves

For the VaR calculation, the yield curves of all historical days are evaluated with the function `interpolateFromData`. At the boundaries it behaves differently from `interpolateRate`:

$$r(t) = \begin{cases} -\dfrac{1}{t}\ln\!\left(1 + \left(e^{-r_1\tau_1} - 1\right)\dfrac{t}{\tau_1}\right), & t < \tau_1 \quad \text{(linear in the discount factor)}, \\ r_j + \dfrac{r_{j+1} - r_j}{\tau_{j+1} - \tau_j}\,(t - \tau_j), & \tau_j < t < \tau_{j+1}, \\ r_j, & t = \tau_j \text{ (exact pillar)}, \\ r_m, & t > \tau_m. \end{cases} \tag{3}$$

For $t < \tau_1$ the discount factor is interpolated linearly between $P(0) = 1$ and $P(\tau_1)$, $P(t) = 1 + (P(\tau_1) - 1)\,t/\tau_1$, and the implied zero rate is derived from it.

Note

For maturities below the first pillar τ_1, (2) and (3) differ: valuation uses a flat zero rate, risk measurement a linearly interpolated discount factor. This matters only if payment dates fall before τ_1 (e.g. $\tau_1 = 1$ year and $T_0 + \delta < 1$).

3 Valuation of the Forward Starting Swap

3.1 Payment schedule

The number of payments and the payment dates are

$$n = \lfloor T \cdot f \rfloor, \qquad t_i = T_0 + i\,\delta, \quad i = 1, \ldots, n. \tag{4}$$

If $T \cdot f$ is not an integer, the incomplete stub period is dropped. Accrual period i runs from t_{i-1} to t_i with $t_0 := T_0$.

3.2 Forward rates

The simple (linear) forward rate for period $[t_{i-1}, t_i]$ follows from no-arbitrage:

$$F_i = \frac{1}{t_i - t_{i-1}}\left(\frac{P(t_{i-1})}{P(t_i)} - 1\right) = \frac{1}{\delta}\left(\frac{P(t_{i-1})}{P(t_i)} - 1\right). \tag{5}$$

3.3 Cash flows and present value

Cash flows of period i

$$\text{Fixed cash flow:} \quad X_i^{\text{fix}} = N\,K\,\delta, \tag{6}$$

$$\text{Floating cash flow:} \quad X_i^{\text{float}} = N\,F_i\,\delta, \tag{7}$$

$$\text{Net cash flow (payer):} \quad C_i = X_i^{\text{float}} - X_i^{\text{fix}} = N\,\delta\,(F_i - K), \tag{8}$$

$$\text{Present value:} \quad \mathrm{PV}_i = C_i\,P(t_i), \qquad \mathrm{PV} = \sum_{i=1}^{n} C_i\,P(t_i). \tag{9}$$

The code computes X_i^{fix} as `notional * fixedRate / frequency` and X_i^{float} as `notional * forwardRate / frequency`; with $\delta = 1/f$ this matches the formulas above.

3.4 Closed form

Proposition 1 (Telescoping floating leg). *By* (5), $X_i^{float}\,P(t_i) = N\big(P(t_{i-1}) - P(t_i)\big)$. *Hence*

$$\mathrm{PV} = N\big(P(T_0) - P(t_n)\big) \;-\; N\,K\,\delta \sum_{i=1}^{n} P(t_i). \tag{10}$$

Proof. $NF_i\delta\,P(t_i) = N\big(P(t_{i-1})/P(t_i) - 1\big)P(t_i) = N\big(P(t_{i-1}) - P(t_i)\big)$; the sum over i telescopes to $N\big(P(t_0) - P(t_n)\big)$. □

Representation (10) serves as a check of the valuation. It also shows that the present value depends on the discount factors at T_0, at all payment dates and (twice) at the final date t_n.

Remark (Par swap rate). The fixed rate for which $\mathrm{PV} = 0$ is $S = \dfrac{P(T_0) - P(t_n)}{\delta \sum_i P(t_i)}$. The calculator does not display S. If $K > S$, the present value of the payer swap is negative.

4 Sensitivities

All sensitivities are obtained by **full revaluation** after a parallel shift of all zero rates by $\Delta r = 1\,\text{bp} = 0.0001$:

$$\mathrm{PV}^{\pm} = \mathrm{PV}\big(r_j \pm \Delta r,\ j = 1, \ldots, m\big).$$

Because the forward rates are recomputed from the shifted curve, $\mathrm{PV}^{\pm}$ contains both the discounting and the projection effect.

4.1 DV01

$$\mathrm{DV01} = -\big(\mathrm{PV}^{+} - \mathrm{PV}\big). \tag{11}$$

A positive DV01 means a loss when rates rise. The present value of a payer swap increases with rates, so its DV01 is typically *negative*.

4.2 Modified duration

Duration is expressed relative to the **notional** N:

$$D = \frac{\mathrm{DV01} \cdot 10\,000}{N} \;\approx\; -\frac{1}{N}\frac{\partial \mathrm{PV}}{\partial r}, \qquad \text{i.e.} \qquad \Delta\mathrm{PV} \approx -D\,N\,\Delta r. \tag{12}$$

If all rates rise by one percentage point, the present value changes by about $-D$ percent of the notional. For a payer swap D is negative, because its value increases when rates rise.

Note: relative to the notional

For swaps the present value is often small relative to the interest rate sensitivity (at inception at market terms PV ≈ 0). A duration relative to the present value, DV01 $\cdot$ 10,000/|PV|, would then become very large and could change sign. Relating it to the notional is common for swaps and gives stable figures even when PV ≈ 0. It is, however, not directly comparable to the present-value-based duration of a bond.

4.3 Convexity

$$\mathcal{C} = \frac{\mathrm{PV}^{+} + \mathrm{PV}^{-} - 2\,\mathrm{PV}}{N \cdot (\Delta r)^2}. \tag{13}$$

The numerator is the central finite-difference approximation of the second derivative with respect to a parallel shift. Like the duration, $\mathcal{C}$ is relative to the notional. Since $N > 0$, the sign is determined by the numerator alone. Together with (12) the second-order approximation is

$$\Delta\mathrm{PV} \approx N\Big(-D\,\Delta r + \tfrac{1}{2}\,\mathcal{C}\,(\Delta r)^2\Big).$$

For a payer swap $\mathcal{C}$ is typically negative, because it *pays* the fixed leg (short position in a fixed-rate cash flow stream).

5 Value at Risk

The VaR is computed with the **variance–covariance (delta-normal) approach**. The risk factors are the log discount factors at the forward start T_0 and at the payment dates; their covariance matrix is estimated from the historical time series.

5.1 Historical discount factor time series

The time series covers D days ($D \geq 251$) with yield curves $\big(r_j^{(k)}\big)$, $k = 1, \ldots, D$. For each day k and each date t_i ($i = 0, \ldots, n$ with $t_0 = T_0$), the zero rate $r^{(k)}(t_i)$ is interpolated with (3) and

$$P_i^{(k)} = \exp\big(-r^{(k)}(t_i)\,t_i\big) \tag{14}$$

is computed. The payment dates t_i are the same for all days (constant maturities).

5.2 Log returns

The risk factors are $X_i = \ln P(t_i)$. Their daily changes are

$$x_i^{(k)} = \ln \frac{P_i^{(k)}}{P_i^{(k-1)}} = -\big(r^{(k)}(t_i) - r^{(k-1)}(t_i)\big)\,t_i, \qquad k = 2, \ldots, D. \tag{15}$$

This yields $M = D - 1$ observations (250 for 251 days). Equation (15) shows that the log return of a discount factor is the negative zero rate change weighted by maturity.

5.3 Covariance and correlation matrix

With the sample means $\bar{x}_i = \frac{1}{M}\sum_k x_i^{(k)}$:

$$\Sigma_{ij} = \frac{1}{M-1}\sum_{k=2}^{D}\left(x_i^{(k)} - \bar{x}_i\right)\left(x_j^{(k)} - \bar{x}_j\right), \tag{16}$$

$$\sigma_i = \sqrt{\Sigma_{ii}}, \qquad \rho_{ij} = \frac{\Sigma_{ij}}{\sigma_i\,\sigma_j}. \tag{17}$$

Two summary figures are also displayed:

$$\text{Avg. volatility:}\quad \bar{\sigma} = \sqrt{\tfrac{1}{n}\textstyle\sum_{i=1}^{n}\Sigma_{ii}} \quad \text{(square root of the mean variance)}, \tag{18}$$

$$\text{Avg. correlation:}\quad \bar{\rho} = \tfrac{1}{n-1}\textstyle\sum_{i=1}^{n-1}\rho_{i,i+1} \quad \text{(adjacent dates only)}. \tag{19}$$

Both are computed over the payment dates $t_1, \ldots, t_n$ only, are informational and do not enter the VaR. With only one payment ($n = 1$), $\bar{\rho}$ is undefined (displayed as `NaN`).

5.4 Sensitivity vector

The floating cash flows themselves depend on the discount factors via (5). By Proposition 1, the present value in terms of the risk factors $X_i = \ln P(t_i)$ with $t_0 := T_0$ is

$$\mathrm{PV} = N\,e^{X_0} - N\,e^{X_n} - NK\delta\sum_{i=1}^{n} e^{X_i}. \tag{20}$$

This gives the full sensitivities $w_i := \partial\mathrm{PV}/\partial X_i$:

$$w_0 = N\,P(T_0), \qquad w_i = -NK\delta\,P(t_i)\ \ (1 \le i < n), \qquad w_n = -N\,P(t_n)\,(1 + K\delta), \tag{21}$$

with $\sum_{i=0}^{n} w_i = \mathrm{PV}$. The VaR thus captures both the fixed *and* the floating leg of the swap. For $T_0 = 0$, $P(T_0) = 1$ is not a risk factor; $w_1, \ldots, w_n$ remain.

5.5 Portfolio variance and VaR

Linearised, $\Delta\mathrm{PV} \approx \sum_i w_i\,\Delta X_i$. Assuming normally distributed log returns with zero mean,

$$\sigma_{\mathrm{PV}}^2 = w^{\top}\Sigma\,w = \sum_{i=0}^{n}\sum_{j=0}^{n} w_i\,\Sigma_{ij}\,w_j, \tag{22}$$

$$\mathrm{VaR}_{\alpha,h} = z_\alpha\;\sigma_{\mathrm{PV}}\;\sqrt{h}. \tag{23}$$

Here $z_\alpha = \Phi^{-1}(\alpha)$ is the α-quantile of the standard normal distribution and h the horizon in days (square-root-of-time scaling). The calculator displays $|\mathrm{VaR}_{\alpha,h}|$.

Confidence level α	95 %	99 %	99.9 %
z_α	1.644854	2.326348	3.090232

5.6 Inverse normal distribution

Φ^{-1} is computed with P. J. Acklam's rational approximation (function `normalInv`). For $p \in [0.02425,\ 0.97575]$, with $q = p - 0.5$ and $s = q^2$,

$$\Phi^{-1}(p) \approx q\,\frac{a_1 s^5 + a_2 s^4 + a_3 s^3 + a_4 s^2 + a_5 s + a_6}{b_1 s^5 + b_2 s^4 + b_3 s^3 + b_4 s^2 + b_5 s + 1},$$

with a corresponding formula in $q = \sqrt{-2\ln p}$ or $q = \sqrt{-2\ln(1-p)}$ in the tails. The relative error is below $1.15 \cdot 10^{-9}$ and negligible for the VaR calculation.

6 Model Assumptions and Limitations

6.1 Assumptions

(A1) A single yield curve is used for discounting and for projecting the floating rates (single-curve approach).

(A2) Zero rates are continuously compounded; time intervals are exact year fractions without calendar or day count convention.

(A3) Daily log returns of the discount factors are jointly normally distributed with zero mean and time-constant covariance; daily returns are independent (square-root-of-time rule).

(A4) All 250 observations are weighted equally (no exponential weighting).

(A5) The linearisation (21) neglects second-order terms.

6.2 Comparison with frozen net cash flows

Earlier versions of the calculator used $w_i = C_i\, P(t_i)$, i.e. they held the net cash flows C_i fixed. The projection risk of the floating leg, which makes up most of a swap's interest rate risk, was then missing, and $P(T_0)$ was not a risk factor. The VaR was therefore far too small.

The example in the user manual (notional EUR 1m, 2Y×3Y, 99 %, 1 day) gives:

Variant	1-day VaR 99 %
Calculator (full sensitivities, 4 risk factors)	EUR 3,480.51
Earlier version (frozen C_i, 3 risk factors)	EUR 52.18
Approximation $z_{0.99} \cdot \lvert\text{DV01}\rvert \cdot \sigma(\Delta r_{3\text{Y}})$	EUR 2,952.55

The calculator's VaR is of the same order as the DV01 approximation. It is somewhat larger because the sensitivities at T_0 and t_n have opposite signs and the rates of the individual maturities do not move perfectly in parallel.

6.3 Further points

- **Short end of the curve:** different extrapolation below τ_1 in valuation and risk measurement (Section 2.3).
- **DV01** measures parallel shifts only; twists and curvature changes appear only in the VaR.
- **Duration and convexity** are relative to the notional and therefore stable even when PV ≈ 0. They are not directly comparable to present-value-based figures.
- **Stub period:** if $T \cdot f$ is not an integer, the incomplete period is ignored.
- **Mean:** the covariance (16) subtracts the means, while the VaR (23) assumes a zero expected value.

7 Mapping Formula – Code

Function	Purpose	Equation
`loadTimeSeries`	read the selected CSV file	–
`importTimeSeriesText`	parse time series, percent → decimal, current curve = last row	–
`loadSampleData`	load a built-in sample series	–
`downloadSample`	save a sample series as CSV	–
`discountFactor`	$P(t) = e^{-rt}$	(1)
`interpolateRate`	interpolation of the current curve	(2)
`interpolateFromData`	interpolation of the historical curves	(3)
`calculateSwapPV`	schedule, forward rates, cash flows, PV	(5)–(10)
`calculateRisk`	DV01, duration, convexity, w, σ_{PV}, VaR	(12), (13), (21)–(23)
`calculateHistoricalCovarianceMatrix`	DF series, log returns, Σ, $\bar{\sigma}$, $\bar{\rho}$	(15), (16)
`buildCorrelationMatrix`	ρ_{ij} from Σ	–
`normalInv`	$\Phi^{-1}(p)$ after Acklam	–
`exportToCSV`	export of all results and matrices	–