Static Leontief Model

User Manual

with Extensions and Sensitivity Analysis

English

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1 Introduction

This tool implements the extended static Leontief model with comprehensive analysis options: breakdown of final use, import analysis, cost structure and value added, price model, satellite accounts (environment and employment) and sensitivity analyses.

1.1 Main Functions

- Basic Leontief model $\mathbf{X} = (\mathbf{E} - \mathbf{A})^{-1}\mathbf{Y}$
- Final use: $\mathbf{Y} = \mathbf{C} + \mathbf{I} + \mathbf{V} + \mathbf{E}$
- Import analysis (up to three import groups)
- Cost structure and value added (wages, profits, taxes, imports)
- Dual price model
- Satellite accounts: CO_2, energy, employment
- Sensitivity analyses with tornado chart
- Export of complete IO tables

1.2 Matrix Sizes

- Inversion of $(\mathbf{E} - \mathbf{A})$ directly in the browser up to 150×150 (e.g. Germany 72×72).
- Beyond that: upload a precomputed inverse as CSV.
- Sensitivity analyses: up to 30×30.

2 Theoretical Foundations

2.1 Basic Model

$$\mathbf{X} = (\mathbf{E} - \mathbf{A})^{-1} \cdot \mathbf{Y} = \mathbf{L} \cdot \mathbf{Y} \tag{1}$$

with the Leontief inverse $\mathbf{L} = (\mathbf{E} - \mathbf{A})^{-1}$. The Neumann series $\mathbf{L} = \mathbf{E} + \mathbf{A} + \mathbf{A}^2 + \ldots$ shows the production rounds.

2.2 Final Use

$$\mathbf{Y} = \mathbf{C} + \mathbf{I} + \mathbf{V} + \mathbf{E} \tag{2}$$

- $\mathbf{C}$: consumption (private households and government)
- $\mathbf{I}$: gross fixed capital formation
- $\mathbf{V}$: changes in inventories (can be negative)
- $\mathbf{E}$: exports

If a dataset contains no $\mathbf{E}$, it is calculated as a residual:

$$\mathbf{E} = \mathbf{Y} - \mathbf{C} - \mathbf{I} - \mathbf{V} \tag{3}$$

Because of linearity, the production effects are $\mathbf{X} = \mathbf{LC} + \mathbf{LI} + \mathbf{LV} + \mathbf{LE}$.

2.3 Import Analysis

Import shares per product group g and sector j (share of output):

$$Im_{g,j} = m_{g,j} \cdot X_j, \qquad Im_j = \sum_{g=1}^{G} Im_{g,j} \tag{4}$$

- $m_{g,j}$: imports of group g per unit of output of sector j (0–1)
- $G \leq 3$ groups; example data: raw materials, intermediate products, services; Destatis: imported intermediate inputs

2.4 Cost Structure and Value Added

Column j of the input-output table describes the costs of sector j:

$$X_j = \underbrace{\sum_{i=1}^{n} a_{ij} X_j}_{\text{intermediate inputs}} + \underbrace{M_j}_{\text{imports}} + \underbrace{L_j + G_j + T_j}_{\text{value added}} \tag{5}$$

In coefficients (shares of output):

$$\sum_{i=1}^{n} a_{ij} + m_j + l_j + g_j + t_j = 1 \tag{6}$$

- Intermediate inputs: column sum of $\mathbf{A}$ times X_j
- m_j: import share (sum of the import groups)
- l_j: wages (compensation of employees)
- g_j: profits (gross operating surplus including depreciation)
- t_j: taxes less subsidies (can be negative)

$$GVA_j = L_j + G_j + T_j \tag{7}$$

Note on Destatis data: besides other taxes on production, t_j also contains taxes on products. $L_j + G_j + T_j$ is therefore somewhat larger than gross value added in the national accounts.

2.5 Price Model (Dual)

$$\mathbf{p}^T = \mathbf{p}^T \mathbf{A} + \mathbf{v}^T \quad \Rightarrow \quad \mathbf{p}^T = \mathbf{v}^T (\mathbf{E} - \mathbf{A})^{-1} \tag{8}$$

- $\mathbf{p}$: price index per sector (base 1)
- $\mathbf{v}$: primary input costs per unit of output

From the databox, $\mathbf{v}$ = share of gross value added is used. Since imported intermediate inputs are not included, $\sum_i a_{ij} + v_j < 1$ and therefore $p_j < 1$. With v_j = value-added share plus import share, $p_j = 1$. For cost changes:

$$\Delta\mathbf{p}^T = \Delta\mathbf{v}^T (\mathbf{E} - \mathbf{A})^{-1} \tag{9}$$

(e.g. a wage increase in one sector). The price multiplier p_j / v_j shows the importance of upstream costs.

2.6 Satellite Accounts: Environment and Employment

$f_{k,j}$ is indicator k per unit of output of sector j:

$$D_{k,j} = f_{k,j} \cdot X_j \qquad \text{(direct)} \tag{10}$$

$$I_{k,j} = \sum_{i=1}^{n} f_{k,i}\, a_{ij}\, X_j \qquad \text{(indirect, first supply stage)} \tag{11}$$

$$T_{k,j} = D_{k,j} + I_{k,j} \qquad \text{(total)} \tag{12}$$

Total intensity over all supply stages: $\mathbf{f}_k^T(\mathbf{E} - \mathbf{A})^{-1}$.

- Example data: CO_2 (t), energy (MWh) and employment (persons employed) per unit of output
- Destatis: persons employed per EUR million of output
- At most three indicators at a time

3 Data

3.1 Databox

All English IO tools access a common databox. It is stored in the browser (localStorage) and is separate for each browser; the German tools use a databox of their own. The example datasets are added automatically when a tool is first opened. Datasets can be exported (JSON, CSV, Excel) and imported in the databox management.

3.2 Example Datasets

Dataset	Final use	Additional vectors
Example 5×5 open economy	C, I, V, E	all (fictitious)
Example 3×3 to 10×10 (ZIP)	C, I; V = E = 0	all (fictitious)
Germany 2022: 72×72, 20×20, 15×15	C, I, V, E	Destatis

Fictitious additional vectors (5×5 and ZIP examples): import shares in three groups (raw materials, intermediate products, services), gross value added, wages, profits and taxes as shares of output, CO_2, energy and employment coefficients. The shares satisfy column sum of $\mathbf{A}$ + imports + wages + profits + taxes = 1.

The example *5×5 open economy* has named sectors (agriculture, industry, energy, construction, services) and allows all extensions. The ZIP examples describe a closed economy ($\mathbf{Y} = \mathbf{C} + \mathbf{I}$), because the dynamic model builds on them.

Destatis data: Federal Statistical Office of Germany (Statistisches Bundesamt), input-output accounts 2022 (tables 81511-03 and 81511-07), Data licence Germany – attribution – version 2.0. Additional vectors: gross value added, wages, profits, taxes, imports (shares), persons employed per EUR million, output; plus an illustrative capital matrix for the dynamic model. Sector names follow the English CPA product classification.

4 User Guide

4.1 Tab: Input

Input method (selection field):

- **Databox** (default): the button `Load data from databox` opens a list of the examples and own datasets. The list closes after the selection.
- **Manual input**: matrix size 3×3 to 6×6, fields for $\mathbf{A}$ and $\mathbf{Y}$.
- **CSV upload**: matrix $\mathbf{A}$, vector $\mathbf{Y}$ and optionally the inverse. Comma, semicolon or tab; decimal comma, header row and sector names are allowed.

Display of a loaded dataset:

- Name, source, sum of $\mathbf{Y}$, scalars (e.g. B_I), note on a capital matrix
- Matrix $\mathbf{A}$ with $\mathbf{Y}$ (from 10 sectors as a 10×10 excerpt)
- Table *Final use and additional vectors per sector*: $\mathbf{Y}$, C, I, V, E and all additional vectors with a sum row; a missing E is calculated and marked
- `Show complete` shows all rows and columns

Then click `Calculate`.

4.2 Tab: Calculation

Shows the Leontief inverse, the output vector $\mathbf{X}$ and the production multipliers.

4.3 Tab: Neumann Series

Approximation $\mathbf{L} \approx \mathbf{E}+\mathbf{A}+\ldots+\mathbf{A}^k$ with convergence chart (tolerance 10^{-4}, at most 50 terms).

4.4 Tab: Extensions

In each step, the button `Load values` loads the matching vectors from the loaded dataset. If it contains none, example values are used. The values are shown in a table (row = sector, column = vector) and can be changed. A calculation in the Input tab is required.

Alternatively, individual vectors can be loaded via `Import CSV` (steps 1–3): n values in one row or listed vertically; comma, semicolon or tab; decimal comma only with semicolon or tab. The values overwrite the corresponding column of the table; their number must match the dimension.

4.4.1 Step 1: Breakdown of Final Use

Input: columns C, I, V and E; $\mathbf{Y}$ is shown for comparison. If E is missing in the dataset, the column contains the calculated value $\mathbf{Y}-\mathbf{C}-\mathbf{I}-\mathbf{V}$. Without dataset values: example breakdown 60 % consumption, 20 % investment, 2 % inventories, remainder exports. CSV import for C, I, V and E.

Check: $\mathbf{C}+\mathbf{I}+\mathbf{V}+\mathbf{E}=\mathbf{Y}$; a deviation is shown.

Output: table per sector with sums and shares of $\mathbf{Y}$.

4.4.2 Step 2: Imports

Input: number of import groups (1–3) and the import shares $m_{g,j}$. From the databox, all vectors whose name contains "Import" are taken.

Output: output, intermediate inputs, imports per group and sum per sector; stacked bar chart.

4.4.3 Step 3: Value Added and Cost Structure

Prerequisite: step 1 has been calculated.

Input: coefficients for wages, profits, taxes and imports (shares of output). From the databox: vectors with "Wages", "Profits", "Taxes" in their name; imports = sum of all import vectors. CSV import for imports, wages, profits and taxes.

Check: column sum of $\mathbf{A}$ + wages + profits + taxes + imports ≈ 1 per sector (tolerance 0.1); in case of deviation the tool asks for confirmation.

Output: output, intermediate inputs, wages, profits, taxes, imports and GVA per sector; chart.

4.4.4 Step 4: Price Model

Input: vector $\mathbf{v}$; from the databox the share of gross value added.

Calculation: $\mathbf{p}^T = \mathbf{v}^T(\mathbf{E} - \mathbf{A})^{-1}$.

Output: v_j, p_j and price multiplier p_j/v_j per sector; chart.

Interpretation: a high multiplier indicates strong intermediate interdependence. On $p < 1$ see Section 2.5.

4.4.5 Step 5: Satellite Accounts

Input: number of indicators (1–3) and coefficients $f_{k,j}$. From the databox, vectors with "CO2", "Emission", "Energy", "Environment" or "Employment" in their name are taken; the column headers show their names.

Output: direct, indirect and total values per sector, average intensity; stacked bar chart.

4.5 Tab: Sensitivity

Each analysis varies one parameter by $\pm\Delta\,\%$ (default 10 %, at most 50 %) and shows the effect. `Analyse all ...` evaluates all parameters of one kind; `Create tornado chart` ranks them by effect. Available up to 30×30.

Analysis	Varied parameter	Prerequisite
A matrix	coefficient a_{ij}; elasticity $\varepsilon = \frac{\Delta X/X}{\Delta a/a}$	basic calculation
Final use	C, I or E of a sector	step 1
Imports	import share of a group and a sector	step 2
Prices	v_j of a sector	step 4
Environment	coefficient $f_{k,j}$	step 5

4.6 Tab: Export

- `Export all results`: base IO table, table with final use broken down ($\mathbf{Y} = \mathbf{C}+\mathbf{I}+\mathbf{V}+\mathbf{E}$), import groups, value-added components, price and environmental results; one CSV file with sections.
- `Export IO tables only`: only the IO tables.
- `Export sensitivity results` (Sensitivity tab).

5 Interpretation Aids

5.1 Production Multipliers

Column sums of $\mathbf{L}$: total output per unit of final demand of a sector.

Multiplier	Interpretation	Example
< 1.2	weak interdependence	services
$1.2 - 1.8$	medium interdependence	manufacturing
> 1.8	strong interdependence	automotive industry

5.2 Import Ratio

$$q_{Im,j} = \frac{\sum_g Im_{g,j}}{X_j} \tag{13}$$

< 0.2: low, $0.2 - 0.5$: medium, > 0.5: high import dependence.

5.3 Value-Added Ratio

$$q_{GVA,j} = \frac{GVA_j}{X_j} \tag{14}$$

Typical values: services 0.6–0.8; manufacturing 0.3–0.5; extraction of raw materials 0.2–0.4.

5.4 Intensity of the Satellite Accounts

$$\varepsilon_k = \frac{\sum_j T_{k,j}}{\sum_j X_j} \tag{15}$$

Identification of hotspots, e.g. for climate policy or employment effects.

6 Workflow Example

Complete analysis with the example *5×5 open economy*:

1. **Input:** `Load data from databox`, choose the 5×5 example, check the data, `Calculate`.
2. **Step 1:** `Load values` → C, I, V, E taken over → `Calculate`; check sum = Y.
3. **Step 2:** `Load values` → three import groups → `Calculate`; identify import-intensive sectors.
4. **Step 3:** `Load values` → wages, profits, taxes, imports → `Calculate`; cost structure per sector.
5. **Step 4:** `Load values` → **v** → `Calculate`; compare price multipliers.
6. **Step 5:** `Load values` → CO_2, energy, employment → `Calculate`.
7. **Sensitivity:** determine critical coefficients with the tornado chart.
8. **Export:** `Export all results`.

With the Destatis data (15×15 or 20×20), steps 1–5 work in the same way; step 2 then has one import group, step 5 the indicator persons employed.

7 Limitations

- Inversion in the browser up to 150×150; beyond that upload a precomputed inverse.
- Sensitivity analyses up to 30×30.
- Sensitivities are local and linear; recalculate for large changes.
- The fictitious additional vectors of the examples serve for illustration, not for analysing real economies.

8 Troubleshooting

8.1 "Sum does not match" (Step 1)

Problem: $\mathbf{C} + \mathbf{I} + \mathbf{V} + \mathbf{E} \neq \mathbf{Y}$.
Solutions: check rounding errors, dimensions and CSV formats; set E as residual.

8.2 Warning "sum of the coefficients deviates from 1" (Step 3)

The cost shares of a sector do not add up to 1. Check whether imports are missing or shares were entered in per cent instead of as decimals.

8.3 Databox Empty or Examples Missing

The databox is separate for each browser; in private windows it is not stored. Examples can be restored in the databox management.

8.4 Export Does Not Work

1. Check browser settings (download and pop-up blockers).
2. Make sure the desired steps have been calculated.
3. Open the developer console (F12).

9 Best Practices

1. **Validation:** observe the consistency checks after every step.
2. **Documentation:** note data sources and assumptions.
3. **Plausibility:** compare with values from the literature.
4. **Sensitivity:** test robustness.
5. **Backup:** export the databox regularly (JSON or Excel).