Four-Dimensional Input-Output Calculator

Exercises at Three Levels of Difficulty

with Solutions

English

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Table of Contents

1 Notes 2

2 Level 1 — Basics 3

3 Level 2 — Application 4

4 Level 3 — Complex Exercises 6

A Solutions 8

Literature 9

1 Notes

The following exercises are assigned to the five tabs of the input-output calculator and cover the four-dimensional model (Tab 1), the models M1–M9 (Tab 2), partial input-output tables (Tab 3), make-use matrices (Tab 4) and the material-value balance (Tab 5). The solutions are in the appendix; they were checked numerically against the logic of the tool.

Checking in the calculator:

- Tab 1: set the number of goods n and sectors m, *Create tables*, enter the intermediate flow matrix and final demand (or load them as CSV), *Calculate*. The combined indices are ordered by sector: G1-B1, G2-B1, …, Gn-B1, G1-B2, … (B = sector).
- Prices are entered per good *and* sector (p_{ik}, fields G1-B1, G2-B1, …); a good can have different prices in different sectors.
- Tabs 2 and 3 use the data calculated in Tab 1; Tab 4 and Tab 5 have their own input (Tab 5 can take over the data from Tab 1).
- Your own results can be stored in Tab 1 with *Save current result in databox*.

2 Level 1 — Basics

Exercise

Exercise 1.1 (Tab 1 — Gross output)

A good is produced in a sector with the following intermediate deliveries to four uses: 10, 5, 3, 2. Final demand is $Y = 15$. Calculate the gross output X.

Exercise

Exercise 1.2 (Tab 1 — Input coefficient)

The delivery $X_{ijkl} = 12$ serves the production of a use with gross output $X_{jl} = 60$. Calculate the input coefficient a_{ijkl}.

Exercise

Exercise 1.3 (Tab 2 — Assigning the models)

Assign each of the models M1, M5 and M9 to the correct combination of input and output side (both characteristics / good only / sector only). Which of these models are square and therefore uniquely solvable, and what are their dimensions?

Exercise

Exercise 1.4 (Tab 4 — Market share matrix)

A sector k produces goods 1, 2, 3 in the quantities $50, 3, 2$. The (given) outputs of goods are $x = (60,\ 20,\ 10)$. Calculate the corresponding row of the market share matrix D ($D_{ki} = V_{ki}/x_i$).

Exercise

Exercise 1.5 (Tab 5 — Residual production, concept)

Explain in your own words what the residual production column in Table A of the material-value balance represents and why it is needed.

3 Level 2 — Application

Exercise

Exercise 2.1 (Tab 1 — Leontief inverse, 2×2)

Given the coefficient matrix

$$A = \begin{pmatrix} 0.20 & 0.10 \\ 0.15 & 0.30 \end{pmatrix}$$

and final demand $Y = (100,\ 80)^\top$. Calculate the Leontief inverse $L = (I - A)^{-1}$ and the gross output $X = LY$.

Exercise

Exercise 2.2 (Tab 2 — Model M4)

Given the intermediate flow matrix (2 goods, 2 sectors, combined index G1-B1, G2-B1, G1-B2, G2-B2):

$$Z = \begin{pmatrix} 10 & 4 & 6 & 2 \\ 3 & 12 & 1 & 5 \\ 2 & 3 & 15 & 6 \\ 1 & 2 & 4 & 20 \end{pmatrix}.$$

Form the numerator matrix of model M4 (not yet divided by X), i.e. sum over the sector index k on the input side.

Exercise

Exercise 2.3 (Tab 3 — Modified final demand)

A row $(i, k) \in N_1$ has the original final demand $Y = 50$ and additionally delivers 6 and 4 units to two uses belonging to N_2. Calculate Y^*.

Exercise

Exercise 2.4 (Tab 4 — Industry technology)

Given

$$B = \begin{pmatrix} 0.30 & 0.10 \\ 0.05 & 0.40 \end{pmatrix}, \qquad D = \begin{pmatrix} 0.90 & 0.05 \\ 0.10 & 0.85 \end{pmatrix}.$$

Calculate the commodity-by-commodity coefficient matrix $A_{ii} = B\,D$ under the industry technology assumption.

Exercise

Exercise 2.5 (Tab 5 — Calculating the residual production column)

An N_1 row delivers the quantities 4 and 7 to the two goods of group N_2 in sector $l = 1$. Calculate the residual production $\text{Res.prod.}_{ik,1}$ for this sector.

Exercise

Exercise 2.6 (Tab 1 — Valuation per good and sector)

The input coefficient of a delivery of good i from sector k to the use (j, l) is $a_{ijkl} = 0.20$. The prices are $p_{ik} = 2$ for the delivering good (from sector k) and $p_{jl} = 1.6$ for the receiving good (in sector l). Calculate the value coefficient $a^{p}_{ijkl} = a_{ijkl}\, p_{ik}/p_{jl}$. How does the result change if the same good i is delivered from another sector k' at the price $p_{ik'} = 1.6$?

4 Level 3 — Complex Exercises

Exercise

Exercise 3.1 (Tab 1 — Complete model)

Given, for $n = 3$ goods and $m = 2$ sectors (combined index G1-B1, G2-B1, G3-B1, G1-B2, G2-B2, G3-B2):

$$Z = \begin{pmatrix} 15 & 10 & 5 & 8 & 4 & 3 \\ 8 & 20 & 6 & 4 & 9 & 2 \\ 4 & 3 & 12 & 2 & 5 & 7 \\ 6 & 5 & 2 & 25 & 10 & 4 \\ 3 & 8 & 4 & 7 & 28 & 6 \\ 2 & 4 & 6 & 3 & 5 & 18 \end{pmatrix}, \qquad Y = \begin{pmatrix} 100 \\ 110 \\ 70 \\ 80 \\ 95 \\ 105 \end{pmatrix}.$$

Calculate the gross output X, the input coefficient a_{1111} and (with tools of your choice, e.g. the calculator) the Leontief inverse L. Check $X = LY$.

Exercise

Exercise 3.2 (Tab 3 — Partial table with valuation)

Use the data from Exercise 3.1. Form the partial table for $N_1 = \{G1, G3\}$ ($N_2 = \{G2\}$) and calculate Y^* for all four rows of N_1. Then value Y^* with the prices per good and sector

G1-B1	G2-B1	G3-B1	G1-B2	G2-B2	G3-B2
1	1	2	1	1	2.5

(good G3 is more expensive, and more expensive in sector 2 than in sector 1).

Exercise

Exercise 3.3 (Tab 4 — Technology assumptions compared)

Given the make matrix V (sector × good) and the use matrix U (good × sector):

$$V = \begin{pmatrix} 60 & 5 & 3 \\ 4 & 70 & 6 \\ 2 & 4 & 55 \end{pmatrix}, \qquad U = \begin{pmatrix} 15 & 6 & 4 \\ 5 & 25 & 8 \\ 3 & 4 & 20 \end{pmatrix}.$$

First determine the consistent final demand $y = x - U\mathbf{1}$ (with x = column sums of V). Calculate the commodity-by-commodity coefficient matrix A_{ii} under both the industry technology and the commodity technology assumption, and in both cases check the back-calculation $X = (I - A_{ii})^{-1} y$ against x.

Exercise

Exercise 3.4 (Tab 5 — Complete material-value balance)

Again use the data from Exercise 3.1, this time with $N_1 = \{G1, G2\}$, $N_2 = \{G3\}$. Form Table A (all four N_1 rows complete, with residual production columns per sector) and Table B (the same rows aggregated over the sectors, for good G1 and good G2). Check that the X values in Table B equal the sum of the two corresponding sector values from Exercise 3.1. In addition, calculate the sector row of sector 1 in column G1-B1 ($\text{Sector}_k[j,l] = \sum_i p_{ik} X_{ijkl}$, sum over *all* goods) with $p_{ik} = 1$ and with the prices from Exercise 3.2.

A Solutions

Level 1

Solution

1.1: $X = 10 + 5 + 3 + 2 + 15 = 35$.

1.2: $a_{ijkl} = 12/60 = 0.20$.

1.3: M1: both characteristics on both sides $(i, k \to j, l)$. M5: good only on both sides $(i \to j)$. M9: sector only on both sides $(k \to l)$. All three are square and uniquely solvable: M1 is $(nm) \times (nm)$, M5 is $n \times n$, M9 is $m \times m$. The other six models are rectangular.

1.4: $D = (50/60,\ 3/20,\ 2/10) = (0.8333,\ 0.15,\ 0.20)$.

1.5: The residual production column sums up all deliveries of the N_1 goods to the N_2 goods (which are not shown in detail), separately by receiving sector. It is needed so that Table A remains complete as a balance despite leaving out N_2: row sum (within N_1) + residual production + Y gives exactly the original gross output X.

Level 2

Solution

2.1:

$$L = \begin{pmatrix} 1.2844 & 0.1835 \\ 0.2752 & 1.4679 \end{pmatrix}, \qquad X = \begin{pmatrix} 143.12 \\ 144.95 \end{pmatrix}.$$

2.2: Numerator matrix (sum over k; rows G1, G2, columns G1-B1, G2-B1, G1-B2, G2-B2):

$$\begin{pmatrix} 12 & 7 & 21 & 8 \\ 4 & 14 & 5 & 25 \end{pmatrix}.$$

2.3: $Y^* = 50 + 6 + 4 = 60$.

2.4:

$$A_{ii} = B\,D = \begin{pmatrix} 0.28 & 0.10 \\ 0.085 & 0.3425 \end{pmatrix}.$$

2.5: $\text{Res.prod.}_{ik,1} = 4 + 7 = 11$.

2.6: $a^p_{ijkl} = 0.20 \cdot 2/1.6 = 0.25$. For a delivery from sector k' with $p_{ik'} = 1.6$: $a^p_{ijk'l} = a_{ijk'l} \cdot 1.6/1.6 = a_{ijk'l}$ — the value coefficient then equals the quantity coefficient. The same good is thus valued differently depending on the delivering sector.

Level 3

Solution

3.1: Gross output (row sum of Z plus Y):

$$X = (145,\ 159,\ 103,\ 132,\ 151,\ 143)^\top.$$

$a_{1111} = 15/145 = 0.1034$. Spectral radius $\rho(A) = 0.336$. Leontief inverse:

$$L = \begin{pmatrix} 1.1295 & 0.0906 & 0.0751 & 0.0940 & 0.0556 & 0.0384 \\ 0.0795 & 1.1601 & 0.0885 & 0.0582 & 0.0971 & 0.0320 \\ 0.0411 & 0.0340 & 1.1441 & 0.0313 & 0.0557 & 0.0693 \\ 0.0660 & 0.0585 & 0.0432 & 1.2503 & 0.1119 & 0.0503 \\ 0.0412 & 0.0815 & 0.0690 & 0.0910 & 1.2480 & 0.0690 \\ 0.0261 & 0.0417 & 0.0837 & 0.0412 & 0.0576 & 1.1541 \end{pmatrix}.$$

The back-calculation $X = LY$ agrees exactly with the X above.
3.2: Rows of $N_1 = \{G1, G3\}$: G1-B1, G3-B1, G1-B2, G3-B2. Deliveries to N_2 are the columns G2-B1 and G2-B2, e.g. G1-B1: $100 + 10 + 4 = 114$.

$$Y^* = (114,\ 78,\ 95,\ 114)^\top.$$

In value terms (each row (i,k) times p_{ik}: 1, 2, 1, 2.5):

$$Y^*_{\text{value}} = (114,\ 156,\ 95,\ 285)^\top.$$

3.3: Consistent final demand: $x = (66,\ 79,\ 64)$, $g = (68,\ 80,\ 61)$, $y = x - U\mathbf{1} = (41,\ 41,\ 37)$.

$$A_{ii}^{\text{industry}} = \begin{pmatrix} 0.2071 & 0.0837 & 0.0737 \\ 0.0898 & 0.2882 & 0.1454 \\ 0.0531 & 0.0637 & 0.2885 \end{pmatrix}, \qquad A_{ii}^{\text{commodity}} = \begin{pmatrix} 0.2415 & 0.0669 & 0.0591 \\ 0.0487 & 0.3442 & 0.1187 \\ 0.0299 & 0.0245 & 0.3608 \end{pmatrix}.$$

Both back-calculations $(I - A_{ii})^{-1} y$ give exactly $x = (66,\ 79,\ 64)$.
3.4: Table A (rows G1-B1, G2-B1, G1-B2, G2-B2; Z columns G1-B1, G2-B1, G1-B2, G2-B2):

Row	Z (4 columns)	Res.prod. B1	Res.prod. B2	Y	X
G1-B1	$15, 10, 8, 4$	5	3	100	145
G2-B1	$8, 20, 4, 9$	6	2	110	159
G1-B2	$6, 5, 25, 10$	2	4	80	132
G2-B2	$3, 8, 7, 28$	4	6	95	151

Table B (aggregated over the sectors):

Good	Z (4 columns)	Res.prod. B1	Res.prod. B2	X
G1	$21, 15, 33, 14$	7	7	277 $(= 145 + 132)$
G2	$11, 28, 11, 37$	10	8	310 $(= 159 + 151)$

The X values in Table B equal exactly the sum of the corresponding sector values from Exercise 3.1.
Sector row of sector 1, column G1-B1 (deliveries from G1-B1, G2-B1, G3-B1): with $p_{ik} = 1$: $15 + 8 + 4 = 27$; with the prices from Exercise 3.2: $1 \cdot 15 + 1 \cdot 8 + 2 \cdot 4 = 31$.

Literature

P. Fleissner, W. Böhme, H.-U. Brautzsch, J. Höhne, J. Siassi, K. Stark: *Input-Output-Analyse. Eine Einführung in Theorie und Anwendungen* [Input-output analysis. An introduction to theory and applications]. Springer-Verlag, Wien, New York 1993.