Loan Amortisation Calculator

Comprehensive Documentation on the Financial Mathematics of Loan Repayment

English

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1 Introduction

The Loan Amortisation Calculator is a comprehensive web-based tool for calculating and analysing various loan repayment models. The tool offers:

- 5 different calculation modes (tabs)
- Graphical visualisation of all calculations
- Comparison functions for multiple loans
- Export functions (CSV)
- Consideration of discount (Disagio) and premium (Agio)
- Flexible payment intervals and payment methods

1.1 Basic Terminology

Loan amount (S_0)
The nominal amount of the loan

Discount (α) Deduction from the disbursement amount (0–20 %)

Premium (Agio)
Surcharge on the disbursement amount (inverse of discount)

Disbursement Amount actually paid out: $AZ_0 = S_0 \cdot (1 - \alpha)$

Interest rate (i) Periodic interest rate

Remaining debt (S_t)
Outstanding debt after period t

Repayment (T_t) Debt repayment in period t

Interest (Z_t) Interest payment in period t

Annuity (A_t) Total payment in period t: $A_t = T_t + Z_t$

2 Tab 1: Instalment Repayment (Basic)

2.1 Fundamental Principle

With instalment repayment the **repayment amount remains constant**, while interest decreases as the outstanding debt falls. The total payment (annuity) therefore *decreases* over time.

2.2 Mathematical Formulae

2.2.1 Constant repayment per period

$$T_t = \frac{T_{\text{year}}}{m} = \text{constant} \tag{1}$$

where m is the payment interval (1 = annual, 12 = monthly).

2.2.2 Payment in arrears (standard)

Interest is calculated on the outstanding debt at the **start** of the period:

$$Z_t = S_{t-1} \cdot i \tag{2}$$
$$A_t = T_t + Z_t \tag{3}$$
$$S_t = S_{t-1} - T_t \tag{4}$$

2.2.3 Payment in advance

Repayment occurs at the **start** of the period; interest is charged on the reduced debt:

$$S^*_{t-1} = S_{t-1} - T_t \tag{5}$$
$$Z_t = S^*_{t-1} \cdot i \tag{6}$$
$$A_t = T_t + Z_t \tag{7}$$
$$S_t = S^*_{t-1} \tag{8}$$

2.2.4 Total term

$$n = \left\lceil \frac{S_0}{T_t} \right\rceil \tag{9}$$

2.2.5 Total interest

$$Z_{\text{total}} = \sum_{t=1}^{n} Z_t \tag{10}$$

2.3 Worked Example

Parameter	Value
Loan amount	200 000 €
Discount	0 %
Interest rate p.a.	3.5 %
Repayment rate (annual)	10 000 €
Payment interval	Annual
Payment method	In arrears

Result:

- Term: 20 years
- Total interest: 36 750 €
- Total cost: 236 750 €

3 Tab 2: Instalment Repayment with Variations

This tab allows simulation of various scenarios during the loan term.

3.1 Scenario 1: Special Repayments

Additional one-off or recurring repayments alongside the regular instalment.

3.1.1 Formula

In period t with special repayment ST_t:

$$S_t = S_{t-1} - T_t - ST_t \tag{11}$$

3.1.2 Effect

- Shortens the term
- Reduces total interest burden
- The earlier the payment, the greater the effect

3.2 Scenario 2: Repayment-Free Years

During the first n_{free} years only interest is paid – no repayment.

3.2.1 Formula

For $t \leq n_{\text{free}}$:

$$T_t = 0 \tag{12}$$
$$Z_t = S_0 \cdot i \tag{13}$$
$$A_t = Z_t \tag{14}$$
$$S_t = S_0 \quad \text{(constant)} \tag{15}$$

For $t > n_{\text{free}}$: normal instalment repayment resumes.

3.3 Scenario 3: Payment Pause

Complete payment stop for duration d starting from year t_{start}.

3.3.1 Formula

For $t_{\text{start}} \leq t < t_{\text{start}} + d$:

$$T_t = 0 \tag{16}$$
$$A_t = 0 \tag{17}$$
$$Z_t = S_{t-1} \cdot i \tag{18}$$
$$S_t = S_{t-1} + Z_t \quad \text{(interest capitalised!)} \tag{19}$$

3.3.2 Important

The outstanding debt **grows** during the payment pause as interest is deferred!

3.4 Scenario 4: Interest Rate Change

The interest rate changes after year t_{change} to i_{new}.

3.4.1 Formula

$$i_t = \begin{cases} i_{\text{old}} & \text{for } t \leq t_{\text{change}} \\ i_{\text{new}} & \text{for } t > t_{\text{change}} \end{cases} \tag{20}$$

3.5 Scenario 5: Repayment Rate Change

The repayment rate changes after year t_{change} to T_{new}.

3.5.1 Formula

$$T_t = \begin{cases} T_{\text{old}} & \text{for } t \leq t_{\text{change}} \\ T_{\text{new}} & \text{for } t > t_{\text{change}} \end{cases} \tag{21}$$

3.6 Scenario 6: Premium (Agio)

Surcharge on disbursement: the borrower receives more than the loan amount.

3.6.1 Formula

With premium β:

$$AZ_0 = S_0 \cdot (1 + \beta) \tag{22}$$

3.6.2 Effect

- Higher disbursement than loan amount
- Lower effective interest burden
- Opposite of discount

4 Tab 3: Annuity Repayment (Basic)

4.1 Fundamental Principle

With annuity repayment the **total payment (annuity) remains constant**. The repayment portion increases while interest decreases.

4.2 Mathematical Formulae

4.2.1 Constant annuity

$$A_t = A = \text{constant} \tag{23}$$

4.2.2 Payment in arrears

$$Z_t = S_{t-1} \cdot i \tag{24}$$
$$T_t = A - Z_t \tag{25}$$
$$S_t = S_{t-1} - T_t \tag{26}$$

4.2.3 Payment in advance

$$T_t = A - S_{t-1} \cdot i \tag{27}$$
$$S_t = S_{t-1} - T_t \tag{28}$$
$$Z_t = S_t \cdot i \tag{29}$$
$$A_t = T_t + Z_t \tag{30}$$

4.2.4 Annuity formula (from given term n)

$$A = S_0 \cdot \frac{i \cdot q^n}{q^n - 1} \tag{31}$$

where $q = 1 + i$.

4.2.5 Term formula (from given annuity A)

$$n = \frac{\ln\left(\frac{A}{A - S_0 \cdot i}\right)}{\ln(q)} \tag{32}$$

4.3 Two Calculation Modes

4.3.1 Mode 1: Annuity → Term

Given: Annuity **Sought:** Term
Iterative calculation until $S_t \leq 0.01$

4.3.2 Mode 2: Term → Annuity

Given: Term **Sought:** Annuity
Uses the annuity formula above.

4.4 Worked Example

Parameter	Value
Loan amount	200 000 €
Discount	0 %
Interest rate p.a.	3.5 %
Annuity (annual)	12 000 €
Payment interval	Monthly
Payment method	In arrears

Result:

- Term: approx. 23.5 years
- Total interest: approx. 82 000 €
- Repayment share rises from 583 € to 993 € (monthly)

5 Tab 4: Annuity Repayment with Variations

Analogous to Tab 2, but with a constant annuity instead of a constant repayment rate.

5.1 Scenarios

5.1.1 1. Special Repayments

Works identically to Tab 2, but reduces the **term** (not the annuity).

5.1.2 2. Repayment-Free Years

For $t \leq n_{\text{free}}$:

$$T_t = 0 \tag{33}$$

$$A_t = Z_t = S_0 \cdot i \tag{34}$$

5.1.3 3. Payment Pause

Identical to Tab 2: interest is capitalised.

5.1.4 4. Interest Rate Change

After the rate change the annuity remains constant, but the split between interest and repayment changes:

$$T_t = A - S_{t-1} \cdot i_{\text{new}} \tag{35}$$

5.1.5 5. Annuity Change

The annuity itself changes:

$$A_t = \begin{cases} A_{\text{old}} & \text{for } t \leq t_{\text{change}} \\ A_{\text{new}} & \text{for } t > t_{\text{change}} \end{cases} \tag{36}$$

Effect: Higher annuity → shorter term, lower interest burden.

5.1.6 6. Premium

Identical to Tab 2.

6 Tab 5: Loan Comparison

6.1 How it works

Compare up to 4 different loans with different terms side by side.

6.2 Configuration per Loan

Each loan can be configured individually:

- Name (freely selectable)
- Loan type (annuity or instalment repayment)
- Loan amount
- Discount
- Interest rate
- Annuity / repayment rate
- Maximum term
- Payment interval

6.3 Calculated Comparison Figures

For each loan:

- Disbursement amount
- Total interest
- Actual term
- Total cost (incl. discount)

6.4 Best-Option Detection

The system automatically identifies the cheapest option using the criterion:

$$\text{Total cost} = \text{Loan amount} + \text{Total interest} + (\text{Loan amount} - \text{Disbursement}) \tag{37}$$

The best option is visually highlighted (green border, star badge).

6.5 Comparison Table

Shows all loans side by side with type, amount, interest rate, total interest, term, total cost, and the extra cost of other options vs. the best option (absolute and percentage).

Loan	Type	Amount	Rate	Interest	Term	Total
House bank	Annuity	200 000 €	3.5 %	82 000 €	23 yr	282 000 €
Online bank	Annuity	200 000 €	3.2 %	71 500 €	22 yr	271 500 €
KfW	Annuity	200 000 €	2.8 %	58 000 €	21 yr	**258 000 €**
Building soc.	Instalment	200 000 €	3.8 %	95 000 €	20 yr	295 000 €

Table 1: Example comparison: KfW is the best option

6.6 Example Comparison

Interpretation:

- KfW saves 13 500 € vs online bank (5.0 %)
- KfW saves 24 000 € vs house bank (8.5 %)
- KfW saves 37 000 € vs building society (14.3 %)

7 Graphical Visualisation

All tabs 1–4 display two charts after calculation.

7.1 Chart 1: Remaining Debt Development

- Line chart
- X-axis: time in years
- Y-axis: remaining debt in €
- Shows debt reduction over time

Interpretation:

- Steep curve = fast repayment
- Flat curve = slow repayment
- Instalment repayment: linearly decreasing
- Annuity repayment: exponentially decreasing

7.2 Chart 2: Interest vs. Repayment

- Stacked bar chart
- X-axis: time in years
- Y-axis: amount in €
- Red = interest, Green = repayment

Interpretation:

- Shows composition of payments
- Instalment: total decreases (constant repayment)
- Annuity: total constant (interest falls, repayment rises)

7.3 Chart in Tab 5: Cost Comparison

- Stacked bar chart by loan name
- Purple = loan amount, Red = total interest, Yellow = discount

8 Discount and Premium in Detail

8.1 Discount (Disagio)

8.1.1 Definition

A discount (α) is a percentage deduction from the disbursement amount:

$$AZ_0 = S_0 \cdot (1 - \alpha) \tag{38}$$

8.1.2 Example

- Loan amount: 100 000 €
- Discount: 6 %
- Disbursement: 100 000 € × 0.94 = 94 000 €

8.1.3 Effects

1. **Higher effective burden:**

$$\text{Effective burden} = \text{Total interest} + (S_0 - AZ_0) \tag{39}$$

2. **Tax-deductible:** Discount can be spread over the term (commercial use)
3. **Lower initial liquidity requirement:** Less equity needed upfront

8.2 Premium (Agio)

8.2.1 Definition

A premium (β) is a percentage surcharge on the disbursement amount:

$$AZ_0 = S_0 \cdot (1 + \beta) \tag{40}$$

8.2.2 Example

- Loan amount: 100 000 €
- Premium: 2 %
- Disbursement: 100 000 € × 1.02 = 102 000 €

8.2.3 Effects

1. **Lower effective burden:**

$$\text{Effective burden} = \text{Total interest} - (AZ_0 - S_0) \tag{41}$$

2. **Rare in practice:** Mostly in special financing models
3. **Higher initial liquidity:** More capital available upfront

8.3 Comparison

Feature	Discount	Premium
Disbursement	< Loan amount	> Loan amount
Effective burden	Higher	Lower
Frequency	Very common	Rare
Nominal rate	Lower	Higher
Effective rate	Higher	Lower

Table 2: Comparison of discount vs. premium

9 Payment Intervals

The calculator supports four payment intervals.

9.1 Overview

Interval	Periods/year (m)	Periodic rate	Example
Annual	1	$i = \frac{i_{\text{p.a.}}}{1}$	$3.5\,\% \to 3.5\,\%$
Semi-annual	2	$i = \frac{i_{\text{p.a.}}}{2}$	$3.5\,\% \to 1.75\,\%$
Quarterly	4	$i = \frac{i_{\text{p.a.}}}{4}$	$3.5\,\% \to 0.875\,\%$
Monthly	12	$i = \frac{i_{\text{p.a.}}}{12}$	$3.5\,\% \to 0.2917\,\%$

Table 3: Payment intervals and periodic interest rates

9.2 Effects on Interest Burden

More frequent payments lead to:

- **Slightly lower total interest** (faster debt reduction)
- **Shorter term** for a constant rate
- **Better planning** through regular smaller amounts

9.3 Example Comparison

Loan: 200 000 €, interest rate: 3.5 % p.a., repayment: 10 000 € p.a.

Interval	Term	Total interest	Difference
Annual	20.0 years	36 750 €	Base
Semi-annual	20.0 years	36 312 €	−438 €
Quarterly	20.0 years	36 089 €	−661 €
Monthly	20.0 years	35 975 €	−775 €

Table 4: Effect of payment interval

10 Payment Methods

10.1 In Arrears (Standard)

10.1.1 How it works

Payment at the **end** of the period:

1. Period begins with remaining debt S_{t-1}
2. Interest calculated on full remaining debt
3. Payment (annuity) at end of period
4. Remaining debt reduced

10.1.2 Formula

$$Z_t = S_{t-1} \cdot i \tag{42}$$
$$T_t = A - Z_t \tag{43}$$
$$S_t = S_{t-1} - T_t \tag{44}$$

10.2 In Advance

10.2.1 How it works

Payment at the **start** of the period:

1. Repayment at start of period
2. Remaining debt reduced immediately
3. Interest charged on reduced debt

10.2.2 Formula

$$S'_t = S_{t-1} - T_t \tag{45}$$
$$Z_t = S'_t \cdot i \tag{46}$$
$$S_t = S'_t \tag{47}$$

Feature	In arrears	In advance
Payment at	Period end	Period start
Interest on	Full debt	Reduced debt
Total interest	Higher	Lower
Common for	Most loans	Rare
Advantage	Standard	Interest saving

Table 5: Comparison of payment methods

10.3 Comparison

10.4 Example

Remaining debt: 100 000 €, interest rate: 3.5 %, repayment: 10 000 €

In arrears:

- Interest: 100 000 € × 3.5 % = 3 500 €
- Repayment: 10 000 €
- Annuity: 13 500 €
- New remaining debt: 90 000 €

In advance:

- Repayment: 10 000 € (immediately)
- New remaining debt: 90 000 €
- Interest: 90 000 € × 3.5 % = 3 150 €
- Annuity: 13 150 €

Saving: 350 € per period (in this example)

11 Using the Calculator

11.1 General Navigation

1. **Select tab:** Click desired tab (1–5)
2. **Enter parameters:** Fill in all relevant fields
3. **Calculate:** Click "Calculate" button
4. **View results:** Displayed automatically after calculation
5. **Export:** CSV export button appears after calculation

11.2 Tabs 1 & 3: Basic Calculations

11.2.1 Input fields

- Initial debt (€)
- Discount (%)
- Interest rate p.a. (%)
- Repayment rate / annuity (€/year)
- Maximum term (years)
- Payment interval (dropdown)
- Payment method (radio buttons)

11.2.2 Results displayed

- Parameter summary (info box)
- Result cards (loan amount, disbursement, total interest, term)
- 2 charts (remaining debt, interest/repayment)
- Detailed amortisation schedule (table)
- Interpretation (info box)

11.3 Tabs 2 & 4: Variations

11.3.1 Selecting a scenario

1. Open dropdown menu
2. Select desired scenario
3. Scenario-specific fields appear
4. Enter parameters
5. Calculate

11.3.2 Adding special repayments

1. Select scenario "Special repayment(s)"
2. Enter date and amount
3. Click "+ Add special repayment" for additional entries
4. Any number of special repayments possible

11.3.3 Comparison with base case

After calculation a green box shows:

- Interest saving (absolute, €)
- Interest saving (relative, %)
- Term reduction (years)
- Base total interest vs. with variations

11.4 Tab 5: Loan Comparison

11.4.1 Adding loans

1. Click "+ Add loan" button
2. Loan card appears
3. Enter name (text field in header)
4. Configure all parameters
5. Add up to 4 loans

11.4.2 Removing loans

- Individually: "×" button in card header
- All: "Delete all" button

11.4.3 Running the comparison

1. Configure at least 2 loans
2. Click "Calculate all" button
3. Individual results appear in cards
4. Best option highlighted in green (⋆ **BEST OPTION**)
5. Comparison table appears below
6. Comparison chart shows cost breakdown

11.5 Export Functions

11.5.1 CSV export

After each calculation (tabs 1–4):

1. "CSV Export" button appears
2. Click button
3. File downloaded
4. Contains complete amortisation schedule
5. Open with Excel, LibreOffice, etc.

11.5.2 CSV structure

Columns:

- Year/Period
- Remaining debt (start)
- Interest
- Repayment
- Special repayment (if applicable)
- Annuity
- Remaining debt (end)

12 Practical Use Cases

12.1 Case 1: Comparing Mortgage Offers

12.1.1 Scenario

You want to buy a property for 300 000 € and have offers from 3 banks:

- **Bank A:** 3.5 % interest, 2 % discount, annuity 15 000 €/year
- **Bank B:** 3.2 % interest, 0 % discount, annuity 15 000 €/year
- **KfW:** 2.8 % interest, repayment-free 3 years, then annuity 15 000 €/year

12.1.2 Procedure

1. Open Tab 5 (Loan comparison)
2. Create 3 loans with respective parameters
3. Calculate all
4. Best option highlighted automatically
5. Read off extra cost of other options

12.1.3 Expected Result

KfW option likely cheapest despite repayment-free years, due to lowest interest rate.

12.2 Case 2: Optimising a Special Repayment

12.2.1 Scenario

You have a loan of 200 000 € at 3.5 % and expect an inheritance of 50 000 € in 5 years.

12.2.2 Procedure

1. Open Tab 2 (instalment repayment with variations)
2. Enter base parameters
3. Select “Special repayment” scenario
4. Special repayment: year 5, amount 50 000 €
5. Calculate
6. View comparison with base case

12.2.3 Interpretation

- Interest saving: approx. 15 000–20 000 €
- Term reduction: approx. 5–7 years
- The earlier the special repayment, the greater the effect

12.3 Case 3: Simulating an Interest Rate Change

12.3.1 Scenario

Fixed rate expires after 10 years. Currently 3.0 %, expected 4.5 %.

12.3.2 Procedure

1. Open Tab 4 (annuity repayment with variations)
2. Enter base parameters at 3.0 %
3. Select “Interest rate change” scenario
4. Set change after 10 years to 4.5 %
5. Calculate

12.3.3 Interpretation

- Extra cost from rate increase visible
- Extended term apparent
- Helps decide: refinance or not?

12.4 Case 4: Payment Pause During Parental Leave

12.4.1 Scenario

Parental leave planned in 3 years for 2 years.

12.4.2 Procedure

1. Open Tab 2 or Tab 4
2. Select “Payment pause” scenario
3. Start: year 3, duration: 2 years
4. Calculate

12.4.3 Interpretation

- Remaining debt **grows** during the pause!
- Capitalised interest visible
- Extra cost vs. continuous payment
- Decision aid: is a payment pause worthwhile?

13 Technical Notes

13.1 Browser Compatibility

The calculator works in all modern browsers:

- Google Chrome (version 90+)
- Mozilla Firefox (version 88+)
- Microsoft Edge (version 90+)
- Safari (version 14+)
- Opera (version 76+)

13.2 Technologies Used

HTML5 Page structure

CSS3 Styling and layout

JavaScript (ES6+) Calculation logic

Chart.js 4.4.0 Charts and visualisation

13.3 Privacy

- All calculations performed **locally in the browser**
- No data transmitted to any server
- No cookies
- No user tracking
- Inputs not saved (except via CSV export)

13.4 Performance

- Calculations in milliseconds
- Supports up to 600 periods (50 years monthly)
- Charts loaded asynchronously
- Optimised for mobile devices

13.5 Offline Use

The calculator only requires an internet connection for Chart.js (CDN). For offline use:

1. Embed Chart.js locally
2. Save HTML file
3. Open in browser

14 Mathematical Deep Dive

14.1 Effective Annual Rate with Discount

14.1.1 Problem

The nominal rate does not account for discount. The effective rate is higher.

14.1.2 Formula

For discount α and nominal rate i_{nom}:

$$i_{\text{eff}} \approx i_{\text{nom}} + \frac{\alpha}{n} \tag{48}$$

where n is the term in years.

14.1.3 Example

- Nominal rate: 3.5 %
- Discount: 6 %
- Term: 20 years

$$i_{\text{eff}} \approx 3.5\,\% + \frac{6\,\%}{20} = 3.5\,\% + 0.3\,\% = 3.8\,\% \tag{49}$$

14.2 Annuity Repayment: Derivation

14.2.1 Starting point

Constant annuity A over n periods:

$$S_0 = \sum_{t=1}^{n} \frac{A}{q^t} \tag{50}$$

14.2.2 Geometric series

$$S_0 = A \cdot \sum_{t=1}^{n} \frac{1}{q^t} = A \cdot \frac{1/q \cdot \left(1 - (1/q)^n\right)}{1 - 1/q} \tag{51}$$

14.2.3 Rearrangement

$$S_0 = A \cdot \frac{1 - (1/q)^n}{q - 1} \tag{52}$$

$$= A \cdot \frac{q^n - 1}{q^{n+1} - q^n} \tag{53}$$

$$= A \cdot \frac{q^n - 1}{q^n \cdot (q - 1)} \tag{54}$$

$$= A \cdot \frac{q^n - 1}{q^n \cdot i} \tag{55}$$

14.2.4 Final formula

$$\boxed{A = S_0 \cdot \frac{i \cdot q^n}{q^n - 1}} \tag{56}$$

14.3 Remaining Debt after t Periods

14.3.1 Instalment repayment

$$S_t = S_0 - t \cdot T \tag{57}$$

Linearly decreasing.

14.3.2 Annuity repayment

$$S_t = S_0 \cdot q^t - A \cdot \frac{q^t - 1}{i} \tag{58}$$

Exponentially decreasing.

15 Frequently Asked Questions (FAQ)

15.1 General

15.1.1 Why different payment methods?

Payment in arrears is the standard. Payment in advance can save interest and is sometimes offered.

15.1.2 What is better: instalment or annuity repayment?

Instalment repayment:

- Pro: Faster debt reduction, lower total interest
- Con: Higher initial burden

Annuity repayment:

- Pro: Constant payment, better planning
- Con: Higher total interest, slower initial repayment

15.1.3 Is discount worthwhile?

Only if:

- Tax-deductible (commercial use)
- Nominal rate significantly lower
- Long fixed-rate period planned

Always compare the effective rate!

15.2 Calculation Questions

15.2.1 Why is my total interest so high?

Total interest increases with:

- Higher interest rate
- Longer term
- Lower repayment rate
- Discount

15.2.2 How much does a special repayment reduce interest?

Rules of thumb:

- After 5 years: approx. 60–70 % interest saving on the repaid amount
- After 10 years: approx. 40–50 %
- After 15 years: approx. 20–30 %

The earlier, the better!

15.2.3 Why does the remaining debt grow during a payment pause?

Interest is not paid but **capitalised** (added to the debt). In the next period, interest is charged on the increased debt (compound interest effect).

15.3 Technical Questions

15.3.1 Are my data saved?

No. All calculations are local in the browser. No data transmission.

15.3.2 Can I work offline?

Yes, but Chart.js must be embedded locally (currently CDN).

15.3.3 Why does calculation take longer with many periods?

JavaScript calculates each period individually. With 600 periods (50 years monthly), approx. 100–200 ms.

15.3.4 Can I save multiple calculations?

Not in the tool itself. Use CSV export and save locally.

16 Glossary

Premium (Agio)	Surcharge on the disbursement amount
Annuity	Sum of interest and repayment in one period
Annuity repayment	Repayment form with constant annuity
Discount (Disagio)	Deduction from the disbursement amount
Effective rate	Actual rate accounting for all costs
Capitalisation	Addition of unpaid interest to the outstanding debt
In arrears	Payment at the end of the period
Nominal rate	Agreed rate without ancillary costs
Instalment repayment	Repayment form with constant repayment rate
Payment pause	Complete payment stop with interest capitalisation
Remaining debt	Outstanding loan amount still to be repaid
Special repayment	Additional one-off repayment
Repayment	Reduction of the loan principal
Repayment-free	Interest only, no principal repayment
In advance	Payment at the start of the period
Payment interval	Frequency of payments (annual, monthly, etc.)
Interest rate change	Change of interest rate during the term
Fixed-rate period	Period with a fixed interest rate

17 Summary

The Loan Amortisation Calculator is a comprehensive tool for credit analysis.

17.1 5 Calculation Modes

1. **Instalment repayment:** Constant repayment rate
2. **Instalment repayment with variations:** 6 scenarios
3. **Annuity repayment:** Constant annuity, 2 modes
4. **Annuity repayment with variations:** 6 scenarios
5. **Loan comparison:** Up to 4 loans in parallel

17.2 Main Functions

- Detailed amortisation schedules
- Graphical visualisation (Chart.js)
- Comparison functions
- Best-option detection
- CSV export
- Discount/premium support
- Flexible payment intervals
- Payment in arrears / in advance

17.3 Application Areas

- Property financing
- Commercial loans
- Consumer loans
- Debt restructuring
- Financing comparisons
- Special repayment optimisation

17.4 Technical Features

- Web-based (no installation)
- Local calculation (privacy)
- Responsive design
- All modern browsers
- High performance

Good luck with your financial planning!