Annuity Calculator

Documentation

English

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Table of Contents

1 Introduction **3**
1.1 Scope of Functions . . . 3

2 Annuity Future Value **3**
2.1 Fundamentals . . . 3
2.2 Mathematical Formulae . . . 3
2.2.1 Payment in arrears . . . 3
2.2.2 Payment in advance . . . 3
2.3 Input Parameters . . . 4
2.4 Output . . . 4

3 Annuity Present Value **4**
3.1 Fundamentals . . . 4
3.2 Mathematical Formulae . . . 4
3.2.1 Constant interest rate – in arrears . . . 4
3.2.2 Constant interest rate – in advance . . . 4
3.3 Valuation Date . . . 4
3.4 Discounting Methods . . . 5
3.4.1 Constant interest rate . . . 5
3.4.2 Zero-coupon curve . . . 5
3.4.3 Rate types . . . 5
3.5 Input for Zero-Coupon Curve . . . 5
3.6 CSV Upload of the Yield Curve . . . 5
3.7 Example Yield Curves . . . 6
3.8 Output . . . 7

4 Perpetuity **7**
4.1 Fundamentals . . . 7
4.2 Calculation Modes . . . 7
4.2.1 Mode 1: Capital → Rate . . . 7
4.2.2 Mode 2: Rate → Capital . . . 7
4.3 Capital Development . . . 7
4.4 Withdrawal Rate Input . . . 8
4.5 Output . . . 8

5 Dynamic Annuity **8**
5.1 Fundamentals . . . 8
5.2 Growth Types . . . 8
5.2.1 Geometric growth . . . 8
5.2.2 Arithmetic growth . . . 8
5.3 Present Value with Extended Discounting . . . 9
5.4 Input Parameters . . . 9
5.5 Output . . . 9

6 CSV Export **9**
6.1 How it works . . . 9
6.2 Exported Data . . . 9
6.3 File Format . . . 10

6.4 Excel Compatibility 10

7 Technical Details **10**
7.1 Implementation 10
7.2 Calculation Accuracy 10
7.3 User Interface 10

8 Application Examples **11**
8.1 Example 1: Retirement Provision 11
8.2 Example 2: Capital Withdrawal 11
8.3 Example 4: Present Value with Zero Curves 11
8.4 Example 3: Dynamic Annuity with Inflation 12

9 References **12**

10 Glossary **12**

1 Introduction

The Annuity Calculator is a web-based tool for solving various financial mathematics problems in the area of annuity calculations. It enables the calculation of annuity future values, annuity present values, perpetuities and dynamic annuities with extensive parameterisation options.

1.1 Scope of Functions

- Annuity future value calculation
- Annuity present value calculation with flexible discounting
- Perpetuity with capital development
- Dynamic annuities (geometric and arithmetic)
- CSV export of all results
- Interactive parameter entry

2 Annuity Future Value

2.1 Fundamentals

The annuity future value R_n is the accumulated value of all annuity payments at time n (end of term).

2.2 Mathematical Formulae

2.2.1 Payment in arrears

Payment occurs at the **end** of each period:

$$R_n = r \cdot \frac{q^n - 1}{i} \tag{1}$$

2.2.2 Payment in advance

Payment occurs at the **start** of each period:

$$R_n = r \cdot q \cdot \frac{q^n - 1}{i} \tag{2}$$

Legend:

- R_n = annuity future value
- r = payment per period
- i = periodic interest rate
- $q = 1 + i$ = accumulation factor
- n = number of periods

Parameter	Description	Example
Payment	Amount per period	1 000 €
Interest rate p.a.	Annual interest rate	5 %
Term	Number of years	10 years
Payment interval	Frequency of payments	Annual / Monthly
Payment method	Timing of payment	In arrears / In advance

Table 1: Input parameters – annuity future value

2.3 Input Parameters

2.4 Output

The tool calculates and displays:

- **Future value:** Total value at end of term
- **Total contributions:** Sum of all payments
- **Interest earned:** Difference between future value and contributions
- **Detail table:** Annual development of capital and interest

3 Annuity Present Value

3.1 Fundamentals

The annuity present value R_0 is the discounted value of all future annuity payments at valuation date t.

3.2 Mathematical Formulae

3.2.1 Constant interest rate – in arrears

$$R_0 = r \cdot \frac{q^n - 1}{i \cdot q^n} \tag{3}$$

3.2.2 Constant interest rate – in advance

$$R_0 = r \cdot q \cdot \frac{q^n - 1}{i \cdot q^n} \tag{4}$$

3.3 Valuation Date

The tool allows valuation at any date t:

$$t = t_{\text{years}} + \frac{t_{\text{months}}}{12} \tag{5}$$

Each payment is discounted from its payment date to the valuation date.

3.4 Discounting Methods

3.4.1 Constant interest rate

The simplest method uses a single constant rate i for all maturities:

$$\mathrm{PV}_j = r_j \cdot (1+i)^{-(t_j - t)} \tag{6}$$

where t_j is the payment date and t is the valuation date.

3.4.2 Zero-coupon curve

For each maturity T a specific rate $i(T)$ is used:

$$\mathrm{PV}_j = r_j \cdot \big(1 + i(T_j)\big)^{-T_j} \tag{7}$$

where $T_j = t_j - t$ is the remaining maturity to the payment.
Interpolation: Linear interpolation between defined nodes:

$$i(T) = i(T_1) + \frac{i(T_2) - i(T_1)}{T_2 - T_1} \cdot (T - T_1) \tag{8}$$

Extrapolation: Before the first node its rate applies, after the last node the last rate applies constantly (flat extrapolation). If a curve only reaches 10 years, for example, all later payments are discounted at the 10-year rate.

3.4.3 Rate types

- **p.a. (per annum):** The stated rate applies to one full year and is divided by the payment interval:

$$i_{\text{period}} = \frac{i_{\text{p.a.}}}{m} \tag{9}$$

  where m is the number of payments per year.

- **Corresponds to payment interval:** The stated rate already applies to the period and is not adjusted.

3.5 Input for Zero-Coupon Curve

1. **CSV upload:** upload a file with maturity and rate (Section 3.6)
2. **Manual entry:** dynamic table with any number of rows
3. **Example curve:** choice of four built-in curves (Section 3.7)

3.6 CSV Upload of the Yield Curve

Below the upload field an expandable note “Notes on CSV upload” explains the following rules:

- **Location:** the file must be stored locally on your computer (or a connected drive); download online files first. The file is processed only in the browser and is not transmitted.
- **Structure:** the first row is a header and is skipped. Then one row per node follows: column 1 maturity in years, column 2 zero rate in percent p.a. (`2.5` for 2.5 %, not `0.025`).

- **Separator:** semicolon, comma or tab, detected from the header. With semicolon or tab a decimal comma is also allowed.
- **Order:** arbitrary; the nodes are sorted by maturity.
- **Maturity range:** for long annuities the curve should reach the end of the annuity; otherwise the last rate applies thereafter (flat extrapolation).

Listing 1: Example: zerocurve_EUR.csv

```
Laufzeit;Zinssatz
0.25;3.0115
0.5;3.0996
1.0;3.1977
2.0;3.2978
3.0;3.4007
5.0;3.5092
7.0;3.6000
10.0;3.7064
```

Alternatively market_data.csv (tab "Annuity Present Value" only): header with columns such as `EUR_Zero_3M`, `EUR_Zero_1Y` ... `USD_Zero_10Y` (maturities 3M, 6M, 1Y, 2Y, 3Y, 5Y, 7Y, 10Y), first column date, one row per day. Currency (EUR, USD, GBP, CHF, JPY) and day (first or last) are chosen in the selection fields.

After loading, the calculator shows the number of nodes and all maturity/rate pairs; an incorrect format produces an error message.

3.7 Example Yield Curves

With the method "Example yield curve" (tabs "Annuity Present Value" and "Dynamic Annuity") four curves are available. The nodes are shown as a table; with "Example curve as CSV (template)" each curve can be downloaded in upload format and used as a template for your own data.

Maturity (years)	EUR	USD	CHF	Typical
0.25	3.0115	4.4977	1.4989	2.10
0.5	3.0996	4.5898	1.6065	2.30
1	3.1977	4.6927	1.7112	2.50
2	3.2978	4.8008	1.7972	2.80
3	3.4007	4.8873	1.9035	3.10
5	3.5092	4.9998	2.0008	3.50
7	3.6000	5.1028	2.1058	3.80
10	3.7064	5.1960	2.1923	4.00
15	–	–	–	4.20
20	–	–	–	4.30
30	–	–	–	4.40

Table 2: Built-in example yield curves (zero rates in % p.a.)

The EUR, USD and CHF curves correspond to the files `zerocurve_EUR.csv`, `zerocurve_USD.csv` and `zerocurve_CHF.csv` and range from 3 months to 10 years. The typical curve is illustrative and reaches 30 years.

3.8 Output

- **Present value at date t:** Total of all discounted payments
- **Total payments:** Sum of all future payments (nominal)
- **Discounting:** Difference between total payments and present value
- **Detail table:** Payment date, payment, (zero rate), present value, cumulative

4 Perpetuity

4.1 Fundamentals

A perpetuity is an infinite series of payments. The tool calculates the sustainable withdrawal rate or the required capital for a desired rate, as well as the capital development at deviating rates.

4.2 Calculation Modes

4.2.1 Mode 1: Capital → Rate

Given the initial capital K_0, the sustainable withdrawal rate is sought.

Sustainable rate (in arrears):

$$r_{\text{sust}} = K_0 \cdot i \tag{10}$$

Sustainable rate (in advance):

$$r_{\text{sust}} = K_0 \cdot \frac{i}{1+i} \tag{11}$$

Interpretation:

- Rate = interest rate p.a. ⇒ capital remains constant
- Rate > interest rate p.a. ⇒ capital declines
- Rate < interest rate p.a. ⇒ capital grows

4.2.2 Mode 2: Rate → Capital

Given the desired withdrawal rate r, the required initial capital is sought.

In arrears:

$$K_0 = \frac{r}{i} \tag{12}$$

In advance:

$$K_0 = r \cdot \frac{1+i}{i} \tag{13}$$

4.3 Capital Development

For each period j:

In arrears:

$$K_j = K_{j-1} \cdot (1+i) - r \tag{14}$$

In advance:

$$K_j = (K_{j-1} - r) \cdot (1+i) \tag{15}$$

4.4 Withdrawal Rate Input

The withdrawal rate is entered as a **percentage p.a. of initial capital**:

- Default = annual interest rate
- Step size: 0.1 %
- Automatically adjusted when interest rate changes

4.5 Output

- **Withdrawal rate p.a.:** Actual withdrawal rate in %
- **Sustainable rate p.a.:** Interest rate for constant capital
- **Initial capital**
- **Capital after n years**
- **Development table:** Annual capital, interest, withdrawal

5 Dynamic Annuity

5.1 Fundamentals

Dynamic annuities are payment series with variable payments. The tool supports two growth types.

5.2 Growth Types

5.2.1 Geometric growth

The rate grows at constant growth rate d:

$$r_t = r_0 \cdot (1+d)^t \tag{16}$$

Future value formula (in arrears, $i \neq d$):

$$R_n = r_0 \cdot \frac{(1+i)^n - (1+d)^n}{i-d} \tag{17}$$

Special case $i = d$:

$$R_n = r_0 \cdot n \cdot (1+i)^{n-1} \tag{18}$$

Present value formula (in arrears, $i \neq d$):

$$R_0 = r_0 \cdot \frac{1 - \left(\frac{1+d}{1+i}\right)^n}{i-d} \tag{19}$$

5.2.2 Arithmetic growth

The rate grows by constant amount Δr:

$$r_t = r_0 + t \cdot \Delta r \tag{20}$$

For arithmetic growth the calculation is iterative:

$$R_n = \sum_{j=1}^{n} (r_0 + j \cdot \Delta r) \cdot (1+i)^{n-j} \tag{21}$$

5.3 Present Value with Extended Discounting

The following discounting methods are available, analogous to the standard present value:

1. **Constant (as above):** Uses the rate from the base parameters
2. **Constant (separate value):** Own discounting rate
3. **Zero-coupon curve (CSV)**
4. **Zero-coupon curve (manual)**
5. **Example curve** (EUR, USD, CHF or typical, see Section 3.7)

An additional **valuation date** (years + months) can be specified.

5.4 Input Parameters

Parameter	Description	Example
Initial payment	First payment	1 000 €
Interest rate	Return rate	5 % p.a.
Growth type	Type of growth	Geometric / Arithmetic
Growth rate	For geometric growth	2 %
Constant increment	For arithmetic growth	50 €
Term	Number of years	10 years
Payment interval	Frequency	Annual / Monthly
Calculation type	Target value	Future value / Present value

Table 3: Input parameters – dynamic annuity

5.5 Output

- **Future value / Present value:** Depending on selected calculation type
- **Total payments:** Sum of all payments (nominal)
- **Interest earned / Discounting:** Difference from total value
- **Detail table:**
  - Future value: payment date, payment, capital, interest
  - Present value: payment date, payment, (zero rate), present value, cumulative

6 CSV Export

6.1 How it works

After each calculation a **CSV Export** button appears.

6.2 Exported Data

1. **Detail table:** All rows and columns of the results table
2. **Summary:** The three main results from the result cards

6.3 File Format

- Format: CSV (Comma-Separated Values)
- Delimiter: Semicolon (;)
- Encoding: UTF-8 with BOM
- Decimal separator: Full stop (.)
- File name: `annuity_calculator_[type]_[date].csv`

6.4 Excel Compatibility

Files are optimised for:

- Microsoft Excel
- LibreOffice Calc
- Google Sheets

7 Technical Details

7.1 Implementation

- **Technology:** Standalone HTML with embedded JavaScript and CSS
- **Browser compatibility:** All modern browsers (Chrome, Firefox, Safari, Edge)
- **Offline capable:** Fully usable without internet connection
- **Responsive design:** Works on desktop, tablet and mobile

7.2 Calculation Accuracy

- JavaScript native arithmetic (IEEE 754 double precision)
- Rounding to 2 decimal places for currency amounts
- Linear interpolation for zero-coupon curves
- Iterative calculation for arithmetic growth

7.3 User Interface

- **Tab navigation:** Four main areas (Future Value, Present Value, Perpetuity, Dynamic)
- **Gradient design:** Modern look in purple/violet tones
- **Info boxes:** Contextual help and explanations
- **Real-time validation:** Immediate feedback on inputs

8 Application Examples

8.1 Example 1: Retirement Provision

Question: How much capital will I have after 30 years if I save 500 € per month?

Inputs:

- Tab: Annuity Future Value
- Payment: 500 €
- Interest rate: 4 % p.a.
- Term: 30 years
- Payment interval: Monthly
- Payment method: In arrears

Result: Future value approx. 347 850 €

8.2 Example 2: Capital Withdrawal

Question: I have saved 500 000 €. How much can I withdraw annually at 3 % over 25 years?

Inputs:

- Tab: Annuity Present Value
- Present value: 500 000 € (solve backwards)
- Interest rate: 3 % p.a.
- Term: 25 years
- Payment interval: Annual

Alternative: Perpetuity tab for sustainable withdrawal without capital depletion.

8.3 Example 4: Present Value with Zero Curves

Question: What is the value today of an annuity of 1 000 € per year over 10 years (in arrears) when discounted with different yield curves?

Inputs: tab Annuity Present Value, payment 1 000 €, term 10 years, annual, in arrears, valuation date 0, discounting method “Example yield curve”.

Curve	Zero rate 10Y	PV of last payment	Annuity PV
EUR	3.71 %	694.94 €	8 286.27 €
USD	5.20 %	602.57 €	7 698.26 €
CHF	2.19 %	805.04 €	8 948.27 €
Typical	4.00 %	675.56 €	8 251.68 €

Table 4: Annuity present value per yield curve

The higher the rate level, the lower the present value. Check for the last payment (EUR): $1000 \cdot 1.037064^{-10} = 694.94$ €.

8.4 Example 3: Dynamic Annuity with Inflation

Question: Retirement provision with annually rising payments (inflation adjustment).
Inputs:

- Tab: Dynamic Annuity
- Initial payment: 1 000 €
- Interest rate: 5 % p.a.
- Growth type: Geometric
- Growth rate: 2 %
- Term: 20 years
- Calculation type: Future value

9 References

The mathematical foundations are based on standard financial mathematics textbooks:

- Tietze, J. (2015): *Einführung in die Finanzmathematik*. Springer Spektrum.
- Luderer, B., Nollau, V., Vetters, K. (2015): *Mathematische Formeln für Wirtschaftswissenschaftler*. Springer Gabler.
- Kruschwitz, L. (2014): *Finanzmathematik*. De Gruyter Oldenbourg.

10 Glossary

Accumulation Compounding; calculation of a future value

Present value Discounted value of future payments at today's date

Discounting Calculation of the present value

Effective rate Actual rate accounting for the compounding period

Perpetuity Infinite series of payments

In arrears Payment at the end of the period

Nominal rate Stated rate without consideration of compounding period

Annuity Regular series of payments

In advance Payment at the start of the period

Zero rate Interest rate for a specific maturity from a yield curve

Compound interest Interest charged on accumulated interest